COPYRIGHT RESERVED VKS(H-2) — Math (3)

2021

Time : 3 hours

Full Marks : 100

Pass Marks : 45

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer ***six*** *questions selecting at least* ***one*** *from each Group in which Q. No. 1 is compulsory.*

1. Fill in the blanks of the following : $2 \times 10 = 20$

(a) The sequence $\left\langle \frac{1}{n} \right\rangle$ convergent to __________.

(b) The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is __________.

(c) A positive term series cannot __________.

(d) If $\sum_{n=1}^{\infty} a_n$ is a series of +ve terms such that

$$\underset{n \to \infty}{\text{Lt}} \frac{u_n}{a_{n+1}} = 1 \text{ then } \sum a_n \text{ is } \_\_\_\_\_\_\_\_\_\_.$$

(e) $\lim_{x \to 0} x \sin\frac{1}{x}$ = __________.

(f) $\sum_{n=1}^{\infty} u_n$ is conditionally convergent if $\sum_{n=1}^{\infty} |u_n|$ is __________.

(g) A __________ operation of the set G is a function that assigns each ordered pair of elements of G to an element of G.

(h) Every group of prime order is __________.

(i) Order of permutation group S_n is __________.

(j) A non-zero element 'a' in a commutative ring R is called a zero-divisor if there is a non-zero element b in R such that __________.

Group – A

2. (a) State and prove Dedekind's theorem for real number system.

(b) State and prove Archimedean properties of real numbers. 16

3. (a) Prove that the sequence $\left\langle \frac{2n+7}{3n+2} \right\rangle$ is monotomically increasing, bounded and tends to the lim + $\frac{2}{3}$.

(b) The necessary and sufficient condition for the convergence of a monotomic sequence is that it is bounded. Prove it. 16

4. (a) Prove that if a function is derivable at a point then it is continuous at that point but converse is not true.

(b) Show that the function $f(x) = \begin{cases} px+1 \text{ if } x \geq 1 \\ x^2 + p \text{ if } x < 1 \end{cases}$

is continuous for all values of p and hence find the condition for existence of the derivative at that point. 16

5. (a) Use Langrange's mean value theorem to prove that $1 + x < e^x < 1 + xe^x \; \forall \; x \geq 0$.

(b) State and prove Maclaurian's theorem to expand f(x). 16

Group – B

6. (a) Prove that the geometric series $1 + x + x^2 + x^3 + \cdots\cdots$ to ∞.

(i) Corverges if $-1 < x < 1$

(ii) Diverges if $x \geq 1$

(iii) Oscillates finitely if $x = -1$

(iv) Oscillates infinitely if $x < -1$

(b) Show that if a series $\sum u_n$ is convergent then $\lim\limits_{n \to \infty} u_n = 0$. Is the converse true ? If yes, explain it, if no give an example. 16

7. (a) State and prove Raabe's test.

(b) Test for convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$. 16

8. (a) Show that the series $\sum_{n=1}^{\infty}\left(1-\cos\frac{\pi}{n}\right)$ converges.

(b) State and prove Leibnitz's test on alternating series. 16

Group – C

9. (a) Prove that set of all non-singular matrix of order 2 with real entries forms a non-abelian group under multiplication of matrices.

(b) Define a cyclic group with an example. Prove that every sub group of a cyclic group is cyclic. 16

10. (a) State and prove Cayley's theorem in group theory.

(b) Define factor group and give an example. 16

11. (a) Prove that every field is an integral domain.

(b) Prove that the set T of numbers of the form a + $\sqrt{2}$b with a and b are rational numbers forms a field. 16